ABSTRACT

The thesis is concerned with the model theory of fields with operators. In the first part we define a convenient and broad setting for operators on fields, which generalizes and unifies many notions existing in the literature. This is done using certain ring schemes, which we call coordinate k-algebra schemes. We classify them fully and prove some algebraic and geometric results about the resulting operators. In the second part we deal with the model theory of fields with such operators, which we call $\mathcal{B}$-fields. We are mostly interested in $\mathcal{B}$-fields which are existentially closed, possibly in some restricted category of $\mathcal{B}$-fields. This in particular involves seeking for a model companion, but also is related to so-called pseudo algebraically closed structures. We prove a very general result saying that in many cases being existentially closed (in a generalized sense) is an elementary property. This encompasses, generalizes and simplifies many results from the literature. We study the resulting first-order theories, most importantly we study dividing lines and quantifier elimination.

29 I 2023

Jakub Gogolok