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Dynamics and computability in Geometric Group Theory

Abstract

The thesis is divided into two parts which correspond to two relatively independent areas of geometric group theory.

In the first part we study computable aspects of amenability. We prove a computable version of Hall's harem Theorem and use it to prove a computable versions of Tarski's Alternative Theorem. Moreover we also prove a new version of Hall's Harem theorem where the final matching is realized by a function with additional properties. We apply it to non-amenable computable coarse spaces to obtain a computable version of Schneider's generalization of Whyte's Theorem.

In the second part is devoted to locally elliptic actions of groups on simply connected small cancellation complexes. In particular, we prove that torsion subgroups of groups defined by $C(6)$, $C(4)$–$T(4)$, or $C(3)$–$T(6)$ small cancellation presentations are finite cyclic groups. This follows from a more general result on the existence of fixed points for locally elliptic actions of groups on simply connected small cancellation complexes. We present an application concerning automatic continuity. We also observe that simply connected $C(3)$–$T(6)$ complexes may be equipped with a CAT(0) metric. This allows us to get stronger results on locally elliptic actions in that case. It also implies that the Tits Alternative holds for groups acting on simply connected $C(3)$–$T(6)$ small cancellation complexes with a bound on the order of cell stabilisers.