LETTER IN SUPPORT OF THE HABILITATION OF MICHAŁ MARCINKOWSKI

Michał Marcinkowski received his PhD in 2015 at the University of Wrocław, under the supervision of Światosław Gal. He has held several postdoctoral positions, including at Regensburg University and at Ben Gurion University. He was an adjunct (equivalent to the American assistant professor rank) at the Polish Academy of Science and currently at the University of Wrocław, where he has been since 2020.

The research contributions that make up his application for habilitation have appeared in six papers, each of which has appeared in high quality journals such as Commentarii Mathematici Helvetici, the Journal of Modern Dynamics, and Mathematische Annalen.

1. Description of Research

Below we give a general summary of the context of Marcinkowski's work, and then a summary of his personal contribution to the field.

1.1. General Remarks.

Marcinkowski works in geometric group theory, which is a field that grew out of combinatorial group theory and Lie theory, under the outsized influence of Mikhail Gromov. The general premise of geometric group theory is to investigate groups, especially finitely generated groups, qua geometric objects when equipped with the natural word metric. Some flexibility is required, including a "quasification" of many natural ideas from geometry, analysis, topology, and algebra, so that they become robust concepts in the coarse geometric context of geometric group theory; of primary concern is that definitions do not depend on a particular choice of generators.

Much of Marcinkowski's work, and especially the work submitted for consideration for his habilitation, is concerned with norms on groups and quasi-morphisms.

Norms can give novel perspectives on the geometry of the group, and the norms that Marcinkowski is generally interested ones are *non-proper* norms, which is to say that a ball of finite radius need not be compact in the discrete topology. Of particular interest are norms which are invariant under natural algebraic operations, such as automorphisms or group conjugation.

Quasimorphisms are quasifications of the notion of a homomorphism. Not only are they of intrinsic interest in geometric group theory, but they also play a key role in studying cohomology theories such as bounded cohomology of groups, which in turn are closely related to basic dynamical concepts such as group actions on the circle. Quasimorphisms also play a key role in the theory of stable commutator length.

Marcinkowski's work in these areas has made significant contributions to the following impressively disparate topics:

- The so-called *bq*–dichotomy for free and surface groups.
- The structure of right-angled Artin and Coxeter groups which admit automorphism-invariant word norms of bounded diameter.
- The global geometry of symplectic diffeomorphism groups of surfaces.
- Bounded cohomology, especially in higher dimension.

1.2. Marcinkowski's Contributions.

In the paper [H1], Marcinkowski and his coauthors introduce the *bq*–dichotomy for groups equipped with a bi–invariant norm. The bi–invariant norm arises in groups that are normally finitely generated, which is to say generated by finitely many conjugacy classes. Then, one takes a minimal normal generating set S, and defines a norm as the minimal number of conjugates of elements of S needed to express a particular element. The resulting metric on the group is invariant under both left and right multiplication.

The authors also introduce the cancellation norm on a group, defined as the minimum number of letters that need to be deleted, over all possible representatives of a group element, in order to obtain the identity. The authors prove that the cancellation norm can be effectively computed, and that it is equivalent to the bi–invariant norm.

Now, a group G satisfied the *bq*–dichotomy if every cyclic subgroup is either bounded in the bi-invariant word metric on G, or there exists a homogeneous quasimorphism on G mapping its generator to a nonzero real value. The authors provide a long list of groups satisfying the *bq*–dichotomy. Interestingly, they also show that if the space of homogeneous quasimorphisms on G has dimension at least n then the group $\mathbb{Z}^n$ embeds isometrically into G.

The study of invariant norms is continued in [H2], where Marcinkowski and his coauthors characterize undistorted elements in the automorphism-invariant norm defined by the number of simple loops needed to represent an element, which makes sense for free and surface groups. They were thus able to answer a question of Calegari, as well as a question of Abért on the existence of many automorphism-invariant quasimorphisms on a free group of rank two.

In [H3], Marcinkowski and Brandenbursky turn their attention to diffeomorphism groups. For a compact, orientable, connected Riemann surface, they prove the existence of infinite dimensional vector space of homogeneous quasimorphisms which are Lipschitz–controlled by the topological entropy. The authors show that area preserving diffeomorphisms can be written as finite products of zero-entropy diffeomorphisms, which allows them to construct an entropy norm, and which they show gives rise to an unbounded, bi–invariant metric. The arguments also apply to the identity components of the area preserving diffeomorphism groups, and to the groups of Hamiltonian symplectomorphisms. The use of configuration spaces and pure braid groups in this paper is particularly remarkable.

In [H4], Marcinkowski studies automorphism invariant norms for right-angled Artin groups and right-angled Coxeter groups, and through a careful analysis of graph products of abelian groups and their automorphisms, he is able to completely characterize boundedness of automorphism invariant word norms on these classes of groups. In particular, the norm is bounded in these cases if and only if the group is free abelian or a direct product of dihedral groups and finite groups of order two.

In [H5], Marcinkowski and Brandenbursky return to diffeomorphism groups of manifolds, studying their bounded cohomology in particular. Let M be a complete Riemannian manifold of finite volume with induced measure μ. Cleverly applying geometric group theory ideas, they are able to construct a map from the bounded cohomology of $\pi_1(M)$ to the bounded cohomology of $\mathrm{Homeo}_0(M, \mu)$ of homeomorphisms preserving μ. They conclude that if $\pi_1(M)$ is acylindrically hyperbolic with trivial center, or if $\pi_1(M)$ surjects to a nonabelian free group, then the third bounded cohomology of $\pi_1(M)$ is infinite dimensional. This is a particularly remarkable result, since so few tools for probing higher bounded cohomology are available. The proof involves a very clever discretization of individual elements of $\mathrm{Homeo}_0(M, \mu)$.

In [H6], Marcinkowski and his coauthors study the L^p geometry of groups of volume preserving diffeomorphisms $\mathrm{Diff}(S, \omega)$, especially for compact, orientable surfaces. Instead of commenting on all the results of the paper, the reviewer would like to highlight one particularly striking corollary: every right-angled Artin group is quasi-isometrically embedded in $\mathrm{Diff}_0(S, \omega)$ in the ℓ_1 metric. This metric is defined as follows: consider a smooth isotopy Φ. Its ℓ_1 length is the time integral of the L^1 norm of its time derivative. The ℓ_1 length of a diffeomorphism is the infimum of lengths taken over all isotopies to the identity.

This corollary is a beautiful illustration of the geometric and algebraic complexity of $\mathrm{Diff}_0(S, \omega)$ that speaks for itself.

1.3. **Other Work.** Marcinkowski has five other publications in various subjects in geometric group theory. These have been written with a number of coauthors, and display a wide range of technical ability and intellectual talent. They include clever counterexamples to well-known conjectures of Burghelea and Dranishnikov. The papers have appeared in high quality journals, such as Groups, Geometry, and Dynamics, the Journal of Topology, and Geometriae Dedicata.

2. General Opinion

Marcinkowski's work occupies a central position in modern geometric group theory, and his work evinces both an admirable technical skill and a far-reaching intellectual vision for the progress of the field. The reviewer notes that Marcinkowski occupies a niche where he is a world expert, and where he is able to attack problems of classical interest through novel and interesting methods. The reviewer is particularly impressed by the papers [H3] and [H6], which relate closely to the reviewer's own work on the structure of pure braid groups on surfaces, and by [H4], as the topic of right-angled Artin groups is close to the reviewer's heart, and because if provides a sharp, easy to comprehend graphical dichotomy regarding the behavior of these groups.

Marcinkowski is clearly an active researcher who has made an impact on a number of subfields of geometric group theory, and he has achieved international prominence. This last claim is supported by the large number of research visits, talks, and citations he has accrued over his career so far. We may all undoubtedly expect more great work from him in the future.

3. Conclusion

Marcinkowski's work and habilitation thesis are of very high quality, and to the reviewer's knowledge comply with all requirements of the relevant statutes of Polish law. The undersigned reviewer endorses Marcinkowski's application without the slightest reservation.

Best regards,

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