Abstract

The goal of this dissertation is to study the existence and stability of nonconstant stationary solutions to certain models from mathematical biology. We analyse two types of models of pattern formation: nonlocal differential equations and diffusion-degenerate reaction-diffusion systems.

In the first part, we prove the existence of nonconstant stationary solutions to a nonlocal and nonlinear model of pattern formation. This model was introduced by the Japanese biologist S. Kondo to explain the mechanism of pattern formation on a skin of Poeciliareticulata fish. We construct two types of stationary solutions by using perturbation techniques and a fixed point argument. We supplement our results with numerical simulations obtained for various model parameters.

Second part of this dissertation is devoted to the analysis of homogeneous diffusion-degenerate reaction-diffusion systems, where single reaction-diffusion equation is coupled with a system of ordinary differential equations. Systems of this type are used to model processes deriving from biology, chemistry or ecology. We study an existence and stability of two types of stationary solutions: regular and jump-discontinuous.

Regular solutions which exhibit continuous dependence between diffusing and non-diffusing components, are shown to exist via perturbation methods. Main result concerning those solutions state that all regular stationary solutions are unstable. Next, we construct jump-discontinuous solutions as a perturbation of constant steady states. Under certain circumstances, such solutions may be stable and we provide sufficient conditions for their linear and nonlinear stability. Since we are working with diffusion-degenerate reaction-diffusion systems, nonlinear stability is not a direct consequence of linear stability and the proof requires additional assumptions.

In the last part of this dissertation, we illustrate our results in the case of diffusion-degenerate reaction-diffusion systems with classical nonlinearities. We construct different types of stationary solutions and determine their stability.