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UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

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Z uwagi na fakt, iż rozprawa doktorska jest w języku angielskim oraz język polski nie jest językiem ojczystym doktoranta proszę o akceptację recenzji w języku angielskim.

REPORT ON THE THESIS
by
JOSUA UNGER

Quantum groups and asymptotic symmetries.

The need for a theory that extends the general relativity and QFT breaking the paradigm of Lorentz and Poincaré symmetries is extensively argued in physics. The point, which is reiterated is that one does not necessarily need to break Lorentz invariance as it is sufficient to replace it with a modified version of symmetries. Fortunately, there exists a vast structure of generalized structures in mathematics, which can enlarge the pool of available symmetries: Hopf algebras and quantum groups.

The thesis has a clear overall structure, first presenting the basics mathematical structure of quantum groups then focusing on the symmetry algebras for a model of (asymptotically) flat spacetime. The original research, based mostly on three papers ([62-64] A.Borowiec, L.Brocki, J.Kowalski-Glikman, J.Unger, *κ-deformed BMS symmetry*, Physics Letters B, 790, 2019; A.Borowiec, L.Brocki, J.Kowalski-Glikman, J.Unger, *BMS algebras in 4 and 3 dimensions, their quantum deformations and duals*, JHEP02 (2021) 084 and A.Borowiec, J.Kowalski-Glikman, J.Unger: *3-dimensional Λ-BMS Symmetry and its Deformations, arXiv:2106.12874)* is contained in Chapter 4 and is mostly technical (with additional computations contained in the Appendix). Chapter 5 contains some new research results on the phenomenological consequences of the structures discussed previously.

Let me briefly discuss the introductory chapters 2-3. The first one is a broad review of all notions that are used later (and even more of them). That includes Poisson Lie Groups, Lie bialgebras, Hopf algebras and quantum deformations and is, for the thesis, more than what is sufficient to understand the rest. For

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example, the h-adic topology is a nice tool to describe the deformations, however, it is not really used later. Section 2.1.7 introducing the R-matrix and notion of quasitriangularity enters an unnecessary part on the intertwiners. It is, however, very good that the definitions are illustrated by relevant examples of the standard quantum deformation, Drinfeld twists, κ-Poincaré algebra and κ-Minkowski.

Chapter 3 covers the idea of asymptotic symmetries starting with a review of asymptotic flatness and Bondi-Metzner-Sachs algebra of supertranslations and superrotattions. This brief review is a good motivation to introduce physically-relevant infinite-dimensional Lie algebras, though the description of possible phenomenology is a bit too superficial. Building upon this, the next section constructs the algebra of charges, which are central extensions of the symmetry algebra, in the case of the BMS algebra leading to the Virasoro algebra as the charge algebra linked to supertranslations. The review is continued for the asymptotic (anti) de Sitter spacetime as well as for the case of 3D gravity, and I find it well-structured and suitable.

Chapter 4 present the original research, published by Josua Unger with the coauthors. The main problem posed here is the classification of the deformations of asymptotic symmetries in 3D and 4D gravity. It starts with the analysis of Witt algebra and the classification of the triangular r-matrices over its subalgebra but up to the automorphisms that extend to the entire Witt algebra. This is illustrated by examples of abelian and Jordanian twists applied to Witt algebras. It is interesting to see the q-version of the abelian twist with equations 4.59-4.66, however, such relations are much better written in a homogeneous form as q-commutation relations. The two last sections discuss deformations of asymptotically flat symmetries in 3D and 4D gravity, using contraction procedure in the 3D case and, again, the know results of classical r-matrices for the Poincaré algebra. My general remark here is that in the addition to precise results (stated as theorems that are proved in the appendix) the rest is based on particular examples of "standard" r-matrices of $o(4,\mathbb{C})$. The author states that "twists corresponding to all r-matrices found can be obtained" but there is no comment what is the problem with studying these deformations. All the theorems, which are significant results are not fully used there.

Chapter 5 is certainly most interesting as it passes to original nonpublished results of the author, with a view to applications and phenomenology. The first task is the construction of the dual algebra and observation that it has some natural finite-dimensional Minkowski-type subalgebras. This works nicely for the abelian and Jordanian twists but rather not for other deformation. Similar computations are presented for the 4-dimensional Jordanian twist (and extended Jordanian) with comparable results. As noncommutativity can lead to some visible physical effects from of modified dispersion relations, it is interesting to see how the discussed types of possible noncommutativity manifest themselves. The analysis of these effects is carried out in section 5.2 and shows that in the

3D case only the extended Jordanian twist can lead to a modified wave equation. The 4D situation is even worse, with a stated conclusion that either there is no deformation of the wave operator or the deformation cannot be consistently performed. A potentially much more interesting physical impact is related to a deformed Lebniz rule and its influence on black hole information loss paradox, following Hawking-Perry-Strominger suggestion (and the critique of that) that it can be related to superrotation charges. Again, here the result of the analysis appears to be rather negative, though the explanation of this fact lacks clarity.

Even though the results are negative this is certainly a good piece of work that allows us to see whether deforming the asymptotic symmetries might bring some observable consequences. I have to admit that to me most of it is rather speculative and would require even much more justification than the standard deformations of the Minkowski space symmetries. The deformation of the asymptotic symmetry algebra alone is a nice step, however, the interpretation of what is the object on which such symmetries are acting or, whether such object exists and has a physical interpretation, is a huge challenge.

Concerning the clarity of the thesis, I have some more specific comments, that I noted during the reading of the manuscript. In particular,

- In the discussion of real forms (2.1.3) of Lie algebras and bialgebras, a paragraph on duality appears – that is not the best place for it, especially that "$*$" is used in both contexts. Much better would have been to add it later, mentioning that Hopf algebra is a self-dual notion.
- In 2.1.7, neither 2.56 nor 2.57 are explained, which would be a good element of the introduction to the rest of the thesis where R-matrices appear.
- In 2.2.1 it should be stressed that the structure of the dual Hopf algebra, the pairing and the action of H on its dual are uniquely determined, so the only ambiguity in the presentation is the choice of generators. It would be also good to explain the Weyl map and 2.120 in detail in this situation and a relation of the Weyl star product to (2.76) (since the same name "star product" is used).
- Section "phenomenology" in 3.1.1 – the "Bondi news" is not properly described (in this context), here the description of the possible observable consequences is not very comprehensive.
- The footnote (4) on p44 is not clear: what means "keeping it real" ?
- In 5.1.1 what is the exact content of (5.1) and (5.2) ?
- There is a gap passing from (5.6) to (5.8). What happens to k on the right-hand side of (5.6)? The same is repeated for the Jordanian twist.
- The explanation of the decoupling of soft and hard modes (end of section 5.3 is not very clear, what does "constraint of being able to define a q-analog" (analogue) means?

There are also some typos and/or omissions, for example:

- p14: "B itself is a left B-module" (B-comodule),
- p33: (3.2) introduces indices A, B that are not explained,
- in 3.1.1, page 35 the formulas 3.14, 3.15 – l_n, T_{mn} are not functions, this is an obvious mistake,
- p55: "in the h-adic topology" – there is no "h" in the formulas,
- (4.41) does not make sense as the sum includes $j=0$,
- (4.99 - 4.103) contain $\Theta_0, \Theta_{\pm 1}$ which are not explained there,
- "cohomolog" (p118) - I think that the correct word is "cohomologous"

Summary of recommendations:

To summarize, I have an overall positive opinion of the Thesis. It contains a good, solid review, placing the research in a physics-motivated background. The main value of the thesis is the hard results (contained in the appendix) and the interesting discussion of possible physical consequences. All these results are significant and prove that the author has not only deep mathematical knowledge, the skill to work out tough problems of mathematical physics but also is inclined to share it with physical insight and has demonstrated research abilities.

The thesis is, despite some minor presentation problems that I pointed out, quite well written and, in my opinion, fulfils all requirements to award a doctoral degree. Therefore, I recommend **the thesis be accepted as the basis to award a doctoral degree in physics.**

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